UPPER LIMIT TO THE GROWTH RATE OF PERTURBATIONS IN RIVLIN-ERICKSEN VISCOELASTIC FLUID IN THE PRESENCE OF ROTATION

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ABSTRACT

The thermal instability of a Rivlin-Ericksen viscoelastic fluid acted upon by uniform vertical rotation and heated from below is investigated. Following the linearized stability theory and normal mode analysis, the paper through mathematical analysis of the governing equations of Rivlin-Ericksen viscoelastic fluid convection with a uniform vertical rotation, for the case of rigid boundaries shows that the complex growth rate σ *of oscillatory perturbations, neutral or unstable for all wave numbers, must lie inside a semi-circle*

$$\sigma_r^{\,2} + \sigma_i^{\,2} \langle \frac{T_A}{(1+\pi^2 F)\left\{1 - \dfrac{T_A F}{\pi^2(1+\pi^2 F)}\right\}} ,$$

in the right half of a complex σ*-plane, where* T_A *is the Taylor number and F is the viscoelasticity parameter, which prescribes the upper limits to the complex growth rate of arbitrary oscillatory motions of growing amplitude in a rotatory Rivlin-Ericksen viscoelastic fluid heated from below. Further, It is established that the existence of oscillatory motions of growing amplitude in the present configuration, depends crucially upon the magnitude of the non-dimensional number* $\dfrac{\pi^2(1+\pi^2 F)}{T_A F}$*, in the sense so long as* $0 \langle \dfrac{\pi^2(1+\pi^2 F)}{T_A F} \le 1$*, no such motions are possible, and in particular PES is valid.*

Key Words: *Thermal convection; Rivlin-Ericksen Fluid; Rotation; PES; Rayleigh number; Taylor number.*
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INTRODUCTION

Stability of a dynamical system is closest to real life, in the sense that realization of a dynamical system depends upon its stability. Right from the conceptualizations of turbulence, instability of fluid flows is being regarded at its root. The thermal instability of a fluid layer with maintained adverse temperature gradient by heating the underside plays an important role in Geophysics, interiors of the Earth, Oceanography and Atmospheric Physics, and has been investigated by several authors (e.g., Bénard[1], Rayleigh[2], Jeffreys[3]) under different conditions. A detailed account of the theoretical and experimental study of the onset of Bénard

Convection in Newtonian fluids, under varying assumptions of hydrodynamics and hydromagnetics, has been given by Chandrasekhar[4]. The use of Boussinesq approximation has been made throughout, which states that the density changes are disregarded in all other terms in the equation of motion except the external force term. There are many elastic-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's[5] constitutive relations. Two such classes of fluids are Rivlin-Ericksen's and Walter's (model B') fluids. Rivlin-Ericksen[6] have proposed a theoretical model for such one class of elastic-viscous fluids.

Bhatia and Steiner[7] have considered the effect of uniform rotation on the thermal instability of a viscoelastic (Maxwell) fluid and found that rotation has a destabilizing influence in contrast to the stabilizing effect on Newtonian fluid. The thermal instability of a Maxwell fluid in hydromagnetics has been studied by Bhatia and Steiner[8]. They have found that the magnetic field stabilizes a viscoelastic (Maxwell) fluid just as the Newtonian fluid. Sharma[9] has studied the thermal instability of a layer of viscoelastic (Oldroydian) fluid acted upon by a uniform rotation and found that rotation has destabilizing as well as stabilizing effects under certain conditions in contrast to that of a Maxwell fluid where it has a destabilizing effect. In another study Sharma[10] has studied the stability of a layer of an electrically conducting Oldroyd fluid[5] in the presence of magnetic field and has found that the magnetic field has a stabilizing influence.

Sharma and kumar[11] have studied the effect of rotation on thermal instability in Rivlin-Ericksen elastico-viscous fluid and found that rotation has a stabilizing effect and introduces oscillatory modes in the system. Kumar et al. [12] considered effect of rotation and magnetic field on Rivlin-Ericksen elastico-viscous fluid and found that rotation has stabilizing effect where as magnetic field has both stabilizing and destabilizing effects. A layer of such fluid heated from below or under the action of magnetic field or rotation or both may find applications in geophysics, interior of the Earth, Oceanography, and the atmospheric physics.

Pellow and Southwell[13] proved the validity of PES for the classical Rayleigh-Bénard convection problem. Banerjee et al[14] gave a new scheme for combining the governing equations of thermohaline convection, which is shown to lead to the bounds for the complex growth rate of the arbitrary oscillatory perturbations, neutral or unstable for all combinations of dynamically rigid or free boundaries and, Banerjee and Banerjee[15] established a criterion on characterization of non-oscillatory motions in hydrodynamics which was further extended by Gupta et al. [16]. However no such result existed for non-Newtonian fluid configurations, in general and in particular, for Rivlin-Ericksen viscoelastic fluid configurations. Banyal[17] have characterized the non-oscillatory motions in couple-stress fluid.

Keeping in mind the importance of non-Newtonian fluids, the present paper is an attempt to prescribe the upper limits to the complex growth rate of arbitrary oscillatory motions of growing amplitude, in a layer of incompressible Rivlin-Ericksen viscoelastic fluid heated from below in the presence of uniform vertical rotation opposite to force field of gravity, when the bounding surfaces are of infinite horizontal extension, at the top and bottom of the fluid are rigid.

FORMULATION OF THE PROBLEM AND PERTURBATION EQUATIONS

Considered an infinite, horizontal, incompressible electrically conducting Rivlin-Ericksen viscoelastic fluid layer, of thickness d, heated from below so that, the temperature and density at the bottom surface z = 0 are T_0 and ρ_0 and at the upper surface z = d are T_d and ρ_d respectively, and that a uniform adverse temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is maintained. The fluid is acted upon by a uniform vertical rotation $\vec{\Omega}(0,0,\Omega)$, parallel to the force field of gravity $\vec{g}(0,0,-g)$.

The equations of motion, continuity and heat conduction, governing the flow of Rivlin-Ericksen viscoelastic fluid in the presence of rotation (Rivlin and Ericksen [6]; Chandrasekhar [4] and Kumar et al [12]) are

$$\frac{\partial \vec{q}}{\partial t}+\left(\vec{q}.\nabla\right)\vec{q}=-\nabla\left(\frac{p}{\rho_o}-\frac{1}{2}\left|\vec{\Omega}\times\vec{r}\right|^2\right)+\vec{g}\left(1+\frac{\delta\rho}{\rho_0}\right)+\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2\vec{q}+2\left(\vec{q}\times\vec{\Omega}\right), \quad (1)$$

$$\nabla.\vec{q}=0, \quad (2)$$

$$\frac{\partial T}{\partial t}+(\vec{q}.\nabla)T=\kappa\nabla^2 T, \quad (3)$$

Where ρ, p, T, ν, ν' and $\vec{q}(u,v,w)$ denote respectively the density, pressure, temperature, kinematic viscosity, kinematic viscoelasticity and velocity of the fluid, respectively and $\vec{r}(x,y,z)$.

The equation of state for the fluid is

$$\rho=\rho_0\left[1-\alpha(T-T_0)\right], \quad (4)$$

Where the suffix zero refer to the values at the reference level z = 0. Here $\vec{g}(0,0,-g)$ is acceleration due to gravity and α is the coefficient of thermal expansion. In writing the equation (1), we made use of the Boussinesq approximation, which states that the density variations are ignored in all terms in the equation of motion except the external force term. The thermal diffusivity κ is assumed to be constant.

The initial state is one in which the velocity, density, pressure, and temperature at any point in the fluid are, respectively, given by

$$\vec{q}=(0,0,0)\,,\ \rho=\rho\ (z),\ \text{p=p(z), T= T(z)}, \quad (5)$$

Assume small perturbations around the basic solution and let $\delta\rho$, δp, θ, and $\vec{q}(u,v,w)$ denote respectively the perturbations in density ρ, pressure p, temperature T and velocity $\vec{q}(0,0,0)$. The change in density $\delta\rho$, caused mainly by the perturbation θ in temperature, is given by

$$\rho+\delta\rho=\rho_0\left[1-\alpha(T+\theta-T_0)\right]=\rho-\alpha\rho_0\theta,\ \text{i.e.}\quad \delta\rho=-\alpha\rho_0\theta. \quad (6)$$

Then the linearized perturbation equations are

$$\frac{\partial \vec{q}}{\partial t} = -\frac{1}{\rho_0}\nabla \delta p - \vec{g}\,\alpha\theta + \left(\nu + \nu' \frac{\partial}{\partial t}\right)\nabla^2 \vec{q} + 2\left(\vec{q} \times \vec{\Omega}\right), \tag{7}$$

$$\nabla . \vec{q} = 0, \tag{8}$$

$$\frac{\partial \theta}{\partial t} = \beta w + \kappa \nabla^2 \theta, \tag{9}$$

Within the framework of Boussinesq approximation, equations (6) - (9), become

$$\frac{\partial}{\partial t}\nabla^2 w = \left(\nu + \nu' \frac{\partial}{\partial t}\right)\nabla^4 w + g\alpha\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) - 2\Omega \frac{\partial \varsigma}{\partial z}, \tag{10}$$

$$\frac{\partial \varsigma}{\partial t} = \left(\nu + \nu' \frac{\partial}{\partial t}\right)\nabla^2 \varsigma + 2\Omega \frac{\partial w}{\partial z}, \tag{11}$$

$$\frac{\partial \theta}{\partial t} = \beta w + \kappa \nabla^2 \theta \tag{12}$$

Where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ and; $\varsigma = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ denote the z-component of vorticity.

NORMAL MODE ANALYSIS

Analyzing the disturbances into normal modes, we assume that the Perturbation quantities are of the form

$$[w, \theta, \varsigma] = [W(z), \Theta(z), Z(z)]\exp\left(ik_x x + ik_y y + nt\right), \tag{13}$$

Where k_x, k_y are the wave numbers along the x- and y-directions, respectively, $k = \left(k_x^{\,2} + k_y^{\,2}\right)^{\frac{1}{2}}$, is the resultant wave number, and n is the growth rate which is, in general, a complex constant. Using (13), equations (10) – (12), in non-dimensional form transform to

$$\left(D^2 - a^2\right)\left[\left(1 + F\sigma\right)\left(D^2 - a^2\right) - \sigma\right]W = Ra^2\Theta + T_A DZ, \tag{14}$$

$$\left[\left(1 + F\sigma\right)\left(D^2 - a^2\right) - \sigma\right]Z = -DW, \tag{15}$$

$$\left(D^2 - a^2 - p_1\sigma\right)\Theta = -W, \tag{16}$$

Where we have introduced new coordinates (x', y', z') = (x/d , y/d, z/d) in new units of length d and $D = d/dz'$. For convenience, the dashes are dropped hereafter. Also we have substituted $a = kd, \sigma = \frac{nd^2}{\nu}$, $p_1 = \frac{\nu}{\kappa}$, is the thermal Prandtl number; $p_2 = \frac{\nu}{\eta}$, is the magnetic Prandtl number; $F = \frac{\nu'}{d^2}$, is the Rilvin-Ericksen kinematic viscoelasticity parameter; $R = \frac{g\alpha\beta d^4}{\kappa\nu}$, is the thermal Rayleigh number and $T_A = \frac{4\Omega^2 d^4}{\nu^2}$, is the Taylor number. Also we have Substituted $W = W_{\oplus}$, $\Theta = \frac{\beta d^2}{\kappa}\Theta_{\oplus}$, $Z = \frac{2\Omega d}{\nu} Z_{\oplus}$, and $D_{\oplus} = dD$, and dropped $(\oplus)$ for convenience.

We now consider the case where both the boundaries are rigid and perfectly conducting and are maintained at constant temperature then the perturbations in the temperature are zero at the boundaries. The appropriate boundary conditions with respect to which equations (14)-(16), must possess a solution are

W = DW = 0, $\Theta = 0$ and Z=0 at z = 0 and z = 1. (17)

Equations (14)-(16), along with boundary conditions (17), pose an eigenvalue problem for σ and we wish to characterize σ_i when $\sigma_r \geq 0$.

We first note that since W and Z satisfy $W(0) = 0 = W(1)$ and $Z(0) = 0 = Z(1)$ in addition to satisfying to governing equations and hence we have from the Rayleigh-Ritz inequality [18]

$$\int_0^1 |DW|^2 dz \geq \pi^2 \int_0^1 |W|^2 dz, \quad (18)$$

And

$$\int_0^1 |DZ|^2 dz \geq \pi^2 \int_0^1 |Z|^2 dz, \quad (19)$$

Further for $W(0) = 0 = W(1)$ and $Z(0) = 0 = Z(1)$, Banerjee et al. [19] have shown that

$$\int_0^1 |D^2W|^2 dz \geq \pi^2 \int_0^1 |DW|^2 dz \text{ And } \int_0^1 |D^2Z|^2 dz \geq \pi^2 \int_0^1 |DZ|^2 dz, \quad (20)$$

MATHEMATICAL ANALYSIS

We prove the following lemma:

Lemma 1: For any arbitrary oscillatory perturbation, neutral or unstable

$$\int_0^1 |Z|^2 dz < \frac{1}{|\sigma|^2} \int_0^1 |DW|^2 dz \text{ and } \int_0^1 \left\{ |DZ|^2 + a^2 |Z|^2 \right\} dz < \frac{1}{\pi^2} \int_0^1 |DW|^2 dz.$$

Proof: Further, multiplying equation (15) with its complex conjugate, and integrating by parts each term on both sides of the resulting equation for an appropriate number of times and making use of boundary condition on Z namely $Z(0) = 0 = Z(1)$ along with (17), we get

$$(1 + 2F\sigma_r + F^2|\sigma|^2) \int_0^1 \left\{ |D^2Z|^2 + 2a^2|DZ|^2 + a^4|Z|^2 \right\} dz + (2\sigma_r + 2F|\sigma|^2) \int_0^1 \left\{ |DZ|^2 + a^2|Z|^2 \right\} dz$$

$$+ |\sigma|^2 \int_0^1 |Z|^2 dz = \int_0^1 |DW|^2 dz, \quad (21)$$

And on utilizing the inequalities (19) and (20), equation (21) gives

$$\int_0^1 |Z|^2 dz < \frac{1}{|\sigma|^2} \int_0^1 |DW|^2 dz \text{ And } \int_0^1 \left\{ |DZ|^2 + a^2|Z|^2 \right\} dz < \frac{1}{\pi^2} \int_0^1 |DW|^2 dz, \quad (22)$$

We prove the following theorem:

Theorem 1: If $R \rangle 0$, $F \rangle 0$, $T_A \rangle 0$, $\sigma_r \geq 0$ and $\sigma_i \neq 0$ then the necessary condition for the existence of non-trivial solution (W, Θ, Z) of equations (16), (17) and (18) together with boundary conditions (19) is that

$$|\sigma|^2 \langle \frac{T_A}{(1+\pi^2 F)\left\{1 - \frac{T_A F}{\pi^2(1+\pi^2 F)}\right\}} .$$

Proof: Multiplying equation (14) by W^* (the complex conjugate of W) throughout and integrating
the resulting equation over the vertical range of z, we get

$$(1+F\sigma)\int_0^1 W^*(D^2-a^2)^2 W dz - \sigma\int_0^1 W^*(D^2-a^2)W dz = Ra^2\int_0^1 W^*\Theta dz + T_A\int_0^1 W^* DZ dz, \quad (23)$$

Taking complex conjugate on both sides of equation (16), we get

$$(D^2 - a^2 - p_1\sigma^*)\Theta^* = -W^*, \quad (24)$$

Therefore, using (24), we get

$$\int_0^1 W^*\Theta dz = -\int_0^1 \Theta(D^2 - a^2 - p_1\sigma^*)\Theta^* dz, \quad (25)$$

Also taking complex conjugate on both sides of equation (15), we get

$$\left[(1+F\sigma^*)(D^2-a^2) - \sigma^*\right]Z^* = -DW^*, \quad (26)$$

Therefore, using (26) and appropriate boundary condition (17), we get

$$\int_0^1 W^* DZ dz = -\int_0^1 DW^* Z dz = \int_0^1 Z\left\{(1+F\sigma^*)(D^2-a^2) - \sigma^*\right\}Z^* dz, \quad (27)$$

Substituting (25) and (27) in the right hand side of equation (23), we get

$$(1+F\sigma)\int_0^1 W^*(D^2-a^2)^2 W dz - \sigma\int_0^1 W^*(D^2-a^2)W dz$$

$$= -Ra^2\int_0^1 \Theta(D^2 - a^2 - p_1\sigma^*)\Theta^* dz + T_A\int_0^1 Z\left\{(1+F\sigma^*)(D^2-a^2) - \sigma^*\right\}Z^* dz, \quad (28)$$

Integrating the terms on both sides of equation (25) for an appropriate number of times by making use of the appropriate boundary conditions (19), along with (17), we get

$$(1+F\sigma)\int_0^1 \left\{|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2\right\}dz + \sigma\int_0^1 \left\{|DW|^2 + a^2|W|^2\right\}dz$$

$$= Ra^2\int_0^1 \left\{|D\Theta|^2 + a^2|\Theta|^2 + p_1\sigma^*|\Theta|^2\right\}dz - T_A(1+F\sigma^*)\int_0^1 \left\{|DZ|^2 + a^2|Z|^2\right\}dz - T_A\sigma^*\int_0^1 |Z|^2 dz, \quad (29)$$

And equating imaginary parts on both sides of equation (29), and cancelling $\sigma_i(\neq 0)$ throughout, we get

$$F\int_0^1 \left\{|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2\right\}dz + \int_0^1 \left\{|DW|^2 + a^2|W|^2\right\}dz$$

$$= -Ra^2 p_1\int_0^1 |\Theta|^2 dz + T_A F\int_0^1 \left\{|DZ|^2 + a^2|Z|^2\right\}dz + T_A\int_0^1 |Z|^2 dz, \quad (30)$$

Now R $\rangle$ 0, F$\rangle$0 and $T_A\rangle$ 0, utilizing the inequalities (19, (20) and (22), the equation (30) gives,

$$\left[(1+\pi^2F)-\left\{\frac{T_AF}{\pi^2}+\frac{T_A}{|\sigma|^2}\right\}\right]\int_0^1|DW|^2dz+I_1\,\langle\,0, \tag{31}$$

Where

$$I_1=F\int_0^1\left\{2a^2|DW|^2+a^4|W|^2\right\}dz+a^2\int_0^1|W|^2dz+Ra^2p_1\int_0^1|\Theta|^2dz\,,$$

Is positive definite.

And therefore, we must have

$$|\sigma|^2\langle\frac{T_A}{(1+\pi^2F)\left\{1-\frac{T_AF}{\pi^2(1+\pi^2F)}\right\}} \tag{32}$$

Hence, if

$$\sigma_r\geq 0\ \text{And}\,\sigma_i\neq 0,\ \text{then}\ |\sigma|^2\langle\frac{T_A}{(1+\pi^2F)\left\{1-\frac{T_AF}{\pi^2(1+\pi^2F)}\right\}}\,.$$

And this completes the proof of the theorem.

In the context of existence of instability in 'oscillatory modes' and that of 'overstability' in the present configuration, we can state prove a theorem as follow:-

Theorem 2: The necessary condition for the existence of instability in 'oscillatory modes' and that of 'overstability' in a Rivlin-Ericksen viscoelastic fluid heated from below, in the presence of uniform vertical rotation is that the Taylor number T_A and the viscoelasticity parameter of the fluid F, must satisfy the inequality $\frac{T_AF}{\pi^2(1+\pi^2F)}\langle 1$, when both the bounding surfaces are rigid

Proof: The inequality (32) for $\sigma_r\geq 0$ and $\sigma_i\neq 0$, can be written as

$$\sigma_r{}^2+\sigma_i{}^2\langle\frac{T_A}{(1+\pi^2F)\left\{1-\frac{T_AF}{\pi^2(1+\pi^2F)}\right\}}\,,$$

We necessarily have,

$$\frac{T_AF}{\pi^2(1+\pi^2F)}\langle 1,$$

This completes the proof.

Presented otherwise from the point of view of existence of instability as stationary convection, the above theorem can be put in the form as follow:-

Theorem 3: The sufficient condition for the validity of the 'exchange principle' and the onset of instability as a non-oscillatory motions of non-growing amplitude in a Rivlin-Ericksen viscoelastic fluid heated from below, in the presence of uniform vertical rotation is that, $\frac{T_AF}{\pi^2(1+\pi^2F)}\rangle 1$, where

T_A is the Taylor number and F is the viscoelasticity parameter, when both the bounding surface are rigid.

or

The onset of instability in a Rivlin-Ericksen viscoelastic fluid heated from below, in the presence of uniform vertical rotation, cannot manifest itself as oscillatory motions of growing amplitude if the Taylor number T_A and the viscoelasticity parameter F, satisfy the inequality $\frac{T_A F}{\pi^2(1+\pi^2 F)} \rangle 1$, when both the bounding surfaces are rigid.

Theorem 4: For stationary convection the Rivlin-Ericksen viscoelastic fluid behaves like an ordinary Newtonian fluid i. e. for $T_A = 0$ implies that $\sigma_r = 0$ and $\sigma_i = 0$, when both the bounding surfaces are rigid.

Proof: The inequality (32), can be written as

$$\sigma_r^{\ 2} + \sigma_i^{\ 2} \langle \frac{T_A}{(1+\pi^2 F)\left\{1 - \frac{T_A F}{\pi^2(1+\pi^2 F)}\right\}},$$

If $T_A = 0$, then we necessarily have,

$\sigma_r = 0$ And $\sigma_i = 0$,

Thus, for stationary convection the Rivlin-Ericksen viscielastic fluid behaves like an ordinary Newtonian fluid, when both the bounding surfaces are rigid and it mathematically establishes the result of Kumar et al [12].

This completes the proof.

CONCLUSIONS

The inequality (32) for $\sigma_r \geq 0$ and $\sigma_i \neq 0$, can be written as

$$\sigma_r^{\ 2} + \sigma_i^{\ 2} \langle \frac{T_A}{(1+\pi^2 F)\left\{1 - \frac{T_A F}{\pi^2(1+\pi^2 F)}\right\}},$$

The essential content of the theorem, from the point of view of linear stability theory is that for the configuration of Rivlin-Ericksen viscoelastic fluid of infinite horizontal extension heated form below, having top and bottom bounding surfaces rigid, in the presence of uniform vertical rotation parallel to the force field of gravity, the complex growth rate of an arbitrary oscillatory motions of growing amplitude, must lie inside a semi-circle in the right half of the $\sigma_r \sigma_i$ - plane whose centre is at the origin and radius is $\sqrt{\frac{\pi^2 T_A}{\pi^2(1+\pi^2 F) - T_A F}}$, where T_A is the Taylor number and F is the viscoelasticity parameter.

Further, it follows from inequality (32) that a sufficient condition for the validity of the 'principle of exchange of stabilities' in rotatory Rivlin-Ericksen viscoelastic fluid convection is that $\frac{T_A F}{\pi^2(1+\pi^2 F)} \rangle 1$. It is therefore clear that the existence of oscillatory motions of growing amplitude in the present configuration depends crucially upon the magnitude of the non-dimensional number $\frac{\pi^2(1+\pi^2 F)}{T_A F}$, in the sense so long as $0 \langle \frac{\pi^2(1+\pi^2 F)}{T_A F} \leq 1$, no such motions are possible, and in particular PES is valid.

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